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STATISTICAL CALIBRATION OF OBSERVING SYSTEMS

Antti A. I. Lange

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STATISTICAL CALIBRATION OF OBSERVING SYSTEMS

Abstract

A fast way of computing discrete Kalman filters (FKF) for Optimum Calibration (O/C) of observing systems has been developed at the Finnish Meteorological Institute (FMI). The main difference as compared with other Kalman Filtering methods is the direct semi-analytical solution of the normal equation system by exploiting its sparse matrix structures with the help of Frobenius' formula. In many cases, it is possible to analyse in real-time very large moving windows of data for improving the observability of calibration parameters and of parameters related to model identification.

The thesis consists of five papers, which are an introductory paper, international patent applications PCT/FI90/00122, PCT/FI93/00192 and PCT/FI96/00621, as well as an article on balloon tracking using Loran-C signal data. The introductory paper outlines the underlying mathematics for a once-only statistical calibration of an observing system. The three PCT publications are hasty priority documents that specify the invented FKF method from three points of view of repeated statistical calibration of different observing systems. It has also turned out that Rao's MINQUE as well as the Horns' and Duncan's AUE fit in these FKF computations. Thus, the operational accuracy of an overdetermined observing system can now be estimated reliably in real-time. Numerical software needed for tracking weather balloons using a hybrid system is disclosed with the fifth paper.

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Preface

The author is very grateful both to the late Swedish Professor of Mathematics at the University of Helsinki, Professor Dr. Gustaf Elfving, who gave the most inspiring introduction to Gauss-Markov's theory on Least Squares Estimation (LSE) methods during his marvellous lectures on mathematical statistics that were given in Swedish and always summarized in Finnish, and to Mr. Reijo Koivu, Captain, M.Sc., the late Head of the Mathematical Research Unit of the Finnish Army, who initiated this reported research on hybrid tracking systems by making available at that time the quite respectable computer resources of an IBM-1410 in 1967.

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The author is in the greatest debt to his wife Terhikki and daughter Saara for their endurance in keeping faith in dad's doings during the past many years. Similarly, the author is also indebted to Messrs. Ensio and Pekka Nieminen of the University of Helsinki for carefully reading and checking the manuscripts as well as to Mrs. Mervi Lindgren and Mr. Esko Lindgren, the Chief of Upper-air Sounding Station 02935 (Tikkakoski, Finland) for collecting the measurement data used in the fifth paper.

April, 1999

Antti Lange

List of abbreviations or acronyms

AKF	Adaptive Kalman Filter
AUE	Almost Unbiased Estimation
BBD	Bordered Block-Diagonal
BLUE	Best Linear Unbiased Estimate
CBA	Canonical Block-Angular
C/I	Covariance Intersection
D/A	Data Assimilation
ECMWF	European Centre for Medium-Range Weather Forecasts
EKF	Extended Kalman Filter
EOF	Empirical Orthogonal Functions
FFT	Fast Fourier Transform
FKF	Fast Kalman Filter for optimum calibration
FMI	Finnish Meteorological Institute
GCM	Global/General Circulation Model
GDOP	Geometric Dilution of Precision
GOS	Global Observing System
GWE	Global Weather Experiment
HPC	High Performance Computing
HWA	Hybrid Windfinding Algorithm
IST	Information Society Technologies
KF	Kalman Filter
KS	Kalman Smoother
LSE	Least Squares Estimation
MATINE	Maanpuolustuksen Tieteellinen Neuvottelukunta
MINQUE	Minimum Norm Unbiased Quadratic Estimation
MLSE	Minimum Least Squares Estimation
NavAid	Navigation Aid
NOAA	U.S. National Oceanic and Atmospheric Administration
NWP	Numerical Weather Prediction
O/C	Optimum Calibration
OT	Optical Theodolite
O/I	Optimum Interpolation
PCT	Patent Cooperation Treaty
PTU	Pressure, Temperature and hUmidity
RT	Radio Theodolite
TWOS	Tropical Wind Observing Ships
UTC	Universal Time Coordinated
WIPO	World Intellectual Property Organization
WLS	Weighted Least Squares
WMO	World Meteorological Organization
WWW	World Weather Watch

CONTENTS

Papers [1] - [5]**:**

[1] Lange, A. (1988): “A high-pass filter for Optimum Calibration of observing systems with applications.” In: Simulation and Optimization of Large Systems, edited by Andrzej J. Osiadacz, Oxford University Press/Clarendon Press, Oxford, pages 311-327. The paper is reprinted by permission of Oxford University Press.

[2] Lange, A. (1990): “Apparatus and method for calibrating a sensor system.” International Application Published under the Patent Co-operation Treaty (PCT), World Intellectual Property Organization, International Bureau, WO 90/13794, PCT/FI90/00122, 15 November 1990 (without international search report, claims and statement).

[3] Lange, A. (1993): “Method for fast Kalman filtering in large dynamic systems.” International Application Published under the Patent Cooperation Treaty (PCT), World Intellectual Property Organization, International Bureau, WO 93/22625, PCT/FI93/00192, 11 November 1993 (without international search report, claims and statement).

[4] Lange, A. (1997): “Method for adaptive Kalman filtering in dynamic systems.” International Application Published under the Patent Co-operation Treaty (PCT), World Intellectual Property Organization, International Bureau, WO 97/18442, PCT/FI96/00621, 22 May 1997 (with amended pages 1, 2, 13 and claim 1).

[5] Lange, A. (1999): “Balloon Tracking using a Hybrid System.” Meteorologisia julkaisuja - Meteorologiska publikationer - Meteorological publications, No. 37, Ilmatieteen laitos - Meteorologiska institutet - Finnish Meteorological Institute, Helsinki, Finland 1999.

1. INTRODUCTION

1.1 Historical background

The statistical theory of Least Squares Estimation (LSE) has been under continuous development since its early foundations were laid by Adrien-M. Legendre (1806) and Karl F. Gauss (1809), who also stated: “But of all these principles ours is the most simple; by others we would be led into the most complicated calculations.” The underlying idea can be characterised as the mathematical method for averaging input data by using a set of optimal weights. The first outcome of the LSE theory was an exact determination of the circulation characteristics of heavenly bodies in our solar system. A related outcome was the Fourier or Spectral Analysis of functions that describe time-dependent physical phenomena. The functions are approximated, in the sense of Weighted Least Squares (WLS), by sums of sinusoidal waves of different wavelengths as discovered by Jean B. Fourier (1822).

Probably one of the last but certainly not the least was the discovery of the Kalman-Bucy (1961) filtering method. It coincided with the advent of modern digital computing and data-communication as Rudolf E. Kalman (1960) published his foundation-laying article: “A new approach to linear filtering and prediction problems.” It represents a technological break-through in Control Theory yielding the present safety of land, marine and aeronautical navigation that is taken so granted today. However, to put it briefly, the theory of optimal linear Kalman filtering provides a provenly stable method for correcting a vector of mean values repeatedly by an optimal exploitation of input data in real-time. This is not always true with other modern control methods such as fuzzy logic.

The purpose of this work is to describe the new method of fast Kalman filtering (FKF) for statistical calibration of observing systems. Nothing new is being added to the mathematical theory as it was when the Fast Fourier Transform (FFT) was discovered by James W. Cooley and John W. Tukey (1965). Similarly, the FKF method makes it now possible to apply the theory of optimal linear Kalman filtering to more sophisticated dynamical systems and make real-time use of extremely long simultaneous time series of observational data. These two advantages are expected to become increasingly important especially to those Information Society Technologies (IST) that are needed for improving public safety. Other similar computing methods have been reported, see e.g. Richard D. Ray (1995). However, these alternative solutions are not as effective as the FKF method, because FKF exploits sparse matrix structures analytically.

1.2 Kalman Filter theory

There is a growing interest in the application of the theory of Kalman Filters (KF) to large dynamical systems like General Circulation Models (GCM) of the atmosphere, and the Global Observing System (GOS) of the World Weather Watch (WWW). The theory covers the whole concept of a Numerical Weather Prediction (NWP) system, see e.g. Fischer et al. (1998). The theory is needed for the proper understanding of forecast error growth and the control of feedback between observations and forecast systems. In fact, there is no principal difference between a sophisticated navigation receiver and an automated weather forecasting system from the point of view of Control Theory.

However, the cases of long- or medium-range weather forecasting are more complex due to highly non-linear connections between GCM systems and observations used. It is not within the scope of this paper to discuss this specific problem area where Kalman Smoothers (KS) are more appropriate than Kalman filtering. This smoothing should be made by using the Adjoint Methods of Variational Calculus instead of the direct inversion methods of locally linear Kalman Filters. The Digital Filtering method of Peter Lynch (1990) may bring new emphasis on the use of Kalman Filters for very short-range weather forecasts, as the resolution of NWP models will probably never match that of ever improving remote-sensing systems like Doppler weather radars and Earth observation satellite systems.

Equations (1) - (3) below describe the underlying Markov process. Equation (1) tells how a measurement vector $\mathbf{y}_t$ of *n* components depends on a state vector $\mathbf{s}_t$ of *m* components and an error vector $\mathbf{e}_t$ of *n* components at time t. This is known as the linearized Measurement (or Observation) Equation:

$$\mathbf{y}_t = H_t\, \mathbf{s}_t + \mathbf{e}_t \quad \text{for } t = 1, 2,... \tag{1}$$

Matrix H_t is the design matrix, usually the Jacobian matrix stemming from the partial derivatives of the actual physical dependencies. The second equation describes the time evolution of an overall system at time t. It is known as the linearized System (or State, or Process) Equation:

$$\mathbf{s}_t = A_t\, \mathbf{s}_{t-1} + B_t\, \mathbf{u}_{t-1} + \mathbf{a}_t \quad \text{for } t = 1, 2,... \text{ (and, } \mathbf{s}_0 \text{ given)} \tag{2}$$

Matrix A_t is the state transition (Jacobian) matrix and B_t is the control/ calibration gain (Jacobian) matrix. Equation (2) tells how a present state vector $\mathbf{s}_t$ of the overall system develops from its previous states $\mathbf{s}_{t-1}$ when it is also affected by control/calibration forcings $\mathbf{u}_{t-1}$ and random noises $\mathbf{a}_t$, see e.g. Abraham et al. (1983).

When measurement errors $\mathbf{e}_t$ and system noises $\mathbf{a}_t$ are neither auto- nor cross-correlated, they do not correlate with $\mathbf{s}_0$ and the covariances are:

$$\left.\begin{aligned} R_t &= \mathrm{Cov}(\mathbf{e}_t) = E(\mathbf{e}_t\,\mathbf{e}_t') \\ Q_t &= \mathrm{Cov}(\mathbf{a}_t) = E(\mathbf{a}_t\,\mathbf{a}_t') \end{aligned}\right\} \tag{3}$$

then the *Kalman forward recursions* from equations (4) - (7) give the Best Linear Unbiased Estimates (BLUE) $\hat{\mathbf{s}}_t$ of present states $\mathbf{s}_t$ as follows:

$$\hat{\mathbf{s}}_t = \tilde{\mathbf{s}}_t + K_t\,(\mathbf{y}_t - H_t\,\tilde{\mathbf{s}}_t) \tag{4}$$

where $\tilde{\mathbf{s}}_t$ is predicted using either an underlying physical model or the State Equation (2) as follows:

$$\tilde{\mathbf{s}}_t = A_t\,\hat{\mathbf{s}}_{t-1} + B_t\,\mathbf{u}_{t-1} \tag{5}$$

and where the error covariance matrices for the prediction (simulation) and the BLUE estimation, respectively, are given as follows:

$$\left.\begin{aligned} \tilde{P}_t &= \mathrm{Cov}(\tilde{\mathbf{s}}_t - \mathbf{s}_t) = E\{(\tilde{\mathbf{s}}_t - \mathbf{s}_t)(\tilde{\mathbf{s}}_t - \mathbf{s}_t)'\} = A_t\,\hat{P}_{t-1}\,A_t' + Q_t \\ \hat{P}_t &= \mathrm{Cov}(\hat{\mathbf{s}}_t - \mathbf{s}_t) = E\{(\hat{\mathbf{s}}_t - \mathbf{s}_t)(\hat{\mathbf{s}}_t - \mathbf{s}_t)'\} = \tilde{P}_t - K_t\,H_t\,\tilde{P}_t \end{aligned}\right\} \tag{6}$$

and where the *Kalman gain matrix* K_t for t = 1, 2,… is computed from:

$$K_t = \tilde{P}_t\,H_t'\,(H_t\,\tilde{P}_t\,H_t' + R_t)^{-1} \tag{7}$$

These recursions (4) - (7) are usually initialized by $\hat{\mathbf{s}}_0 = E\mathbf{s}_0\ (\cong\vec{0})$ and $\hat{P}_0 = \mathrm{Cov}(\hat{\mathbf{s}}_0 - \mathbf{s}_0) = \mathrm{Cov}(\mathbf{s}_0)$, which are assumed to be known (Jazwinski, 1970).

The presented recursive estimation of $\hat{\mathbf{s}}_t$ provides the maximum likelihood adjustments to the mean values given in vector $\hat{\mathbf{s}}_{t-1}$ when errors $\mathbf{e}_t$ and $\mathbf{a}_t$ are normally distributed. The Kalman gain computation in Equation (7) requires the matrix $H_t\tilde{P}_tH_t'+R_t$ of size nxn to be inverted where n is the number of measurements available. This is the best way of updating a mean value vector when only a few measurements are available. However, it is necessary (and sufficient) for the stability of an optimal linear Kalman Filter that its observability and controllability conditions are satisfied (Kalman, 1960). Thus, this number n must be very large (>>m) in order to keep all of the m state (and calibration) parameters observable. This is the computational problem dealt with in papers [2], [3] and [4]. The stability issues of Kalman filtering with unknown error covariances have not been within the scope of this study, see e.g. Julier et al. (1995).

1.3 Statistical calibration

The Swedish National Testing and Research Institute, for example, specifies the word *calibration* as follows: “Calibration of an instrument means determining by how much the instrument reading is in error by checking it against a measurement standard of known error.” The basic idea of this work on statistical *Optimum Calibration* (O/C) does not fit in this definition in a narrow sense.

Statistical calibration is understood here as a real-time method of determining and adjusting errors of instrumental readings by comparing them against optimal combinations of all available data. Meteorologists rather speak of Optimum Interpolation (O/I) and Data Assimilation (D/A), respectively. However, there is no principal controversy between the two different uses of the word “calibration.” The error variance of a linear combination of the instrument readings can now be estimated from their observed internal consistency e.g. by using the Minimum Norm Quadratic Unbiased Estimation (MINQUE) theory, see e.g. Rao (1972 or 1975) and Horn et al. (1975). This has not always been so straightforward due to the very demanding computational requirements and lack of overdetermination.

The statistical calibration based on an optimal linear Kalman Filter is stable under the observability and controllability conditions formulated by Kalman (1960). The stability of calibration can be monitored by predicting or estimating the error covariances $\tilde{P}_t$ or $\hat{P}_t$, respectively, from Equations (6). A non-linear system is handled by the local linear approximations used by an Extended Kalman Filter (EKF). The observability and controllability conditions are still necessary. They are no longer sufficient to guarantee the stability, as predicted error propagation may become grossly misleading. The error covariances can be adjusted by the MINQUE or AUE methods.

A typical problem with statistical calibration of observing systems is that the observability of a calibration parameter is weak or even non-existent. It is severe Geometric Dilution of Precision (GDOP) that usually is the most critical factor of a balloon tracking system. The linearizations made in Equations (1) and (2) ignore all non-linear effects and the synergy of a good sensor combination is lost if the measurements are processed one at a time. Thus, the number *n* of simultaneously analysed measurements should be as large as possible for best possible observability.

Statistical calibration is a viable alternative to physical calibration in many cases where in situ calibration is either too expensive or technically impossible to be used at all. Optimum Calibration (O/C) armed with the FKF computations can analyse most comprehensive time series of data for improving real-time estimation of both the values and the error variances of those calibration parameters that otherwise may not be observable at all.

2. SUMMARIES OF THE PAPERS

This thesis comprises two reprinted papers [1] and [5] as well as three reprinted international patent applications [2], [3] and [4] that describe the FKF solutions of three different problem areas of Kalman filtering. The first paper lays the statistical foundation of *Optimum Calibration* (O/C). The fifth one discloses a practical FKF application. In papers [3] and [4], a general System Equation (2) is modified as follows:

$$A_t \hat{\mathbf{s}}_{t-1} + B_t \ \mathbf{u}_{t-1} = I \ \mathbf{s}_t + A_t (\hat{\mathbf{s}}_{t-1} - \mathbf{s}_{t-1}) - \mathbf{a}_t \tag{8}$$

and combined with the Measurement Equation (1) in order to obtain the so-called Augmented Model:

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{A}_t \hat{\mathbf{s}}_{t-1} + \mathbf{B}_t \mathbf{u}_{t-1} \end{bmatrix} = \begin{bmatrix} H_t \\ I \end{bmatrix} \mathbf{s}_t + \begin{bmatrix} \mathbf{e}_t \\ \mathbf{A}_t (\hat{\mathbf{s}}_{t-1} - \mathbf{s}_{t-1}) - \mathbf{a}_t \end{bmatrix} \tag{9}$$

i.e. $\mathbf{z}_t = Z_t \ \mathbf{s}_t + \eta_t$

The state parameters are estimated by using the well-known solution of a Regression Analysis problem as follows:

$$\hat{\mathbf{s}}_t = (Z_t' \ V_t^{-1} \ Z_t)^{-1} Z_t' \ V_t^{-1} \mathbf{z}_t \tag{10}$$

where $\hat{P}_t = \mathrm{Cov}(\hat{\mathbf{s}}_t - \mathbf{s}_t) = (Z_t' \ V_t^{-1} \ Z_t)^{-1}$
$V_t = \mathrm{Cov}(\eta_t) = E(\eta_t \eta_t')$.

This computational solution is algebraically equivalent to the Kalman recursions (4) - (7). Equation (10) is preferred because the matrix $Z_t'V_t^{-1}Z_t$ to be inverted here is only mxm, where m is the number of all state parameters. This is the approach by Harvey (1981) that is fundamental to the invented FKF method. The size of an “observation and forecast error covariance matrix” V_t in Equation (10) is $(n+m)$x$(n+m)$. Its inversion is known as the problem of “1000 Crays working in tandem” (Gal-Chen, 1988). Thus, these very large matrices V_t (t = 1, 2,…) are initially made block- (or band-) diagonal by using the various methods outlined in papers [1] - [4]. This partial diagonalization of these matrices V_t leads to matrices $Z_t'V_t^{-1}Z_t$ that are *sparse* and have the so-called Bordered Block-Diagonal (BBD) structure. It is this sparsity that makes it possible to invert very large matrices block by block with the help of *Frobenius’ formula*:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} + A^{-1}BH^{-1}CA^{-1} & -A^{-1}BH^{-1} \\ -H^{-1}CA^{-1} & H^{-1} \end{bmatrix} \quad \text{where } H = D - CA^{-1}B$$

as required by the FKF method.

Improved real-time estimates of the forecast error covariances $\tilde{P}_t$ (t = 1, 2,…) are obtained from Equations (6). Observed values of the residuals η_t, i.e. *innovation sequences,* are provided by the FKF computations. Thus, observed internal consistencies of different measurements can be submitted for real-time statistical analyses and testing (Lange, 1983). These tests may detect serious calibration drifts that need immediate built-in-correction. The tedious MINQUE and AUE computations are also speeded up significantly by sparse-matrix partitionings exploited by the FKF method and a further improvement in estimating the "observation and forecast error covariances" V_t can be obtained. However, it has not been within the scope of this reported work to study which error covariance estimates should actually be used for best possible results in Optimum Calibration (O/C).

In paper [1], meteorological observing systems are described from the point of view of statistical modelling. Every sensor system aims at measuring one or only a few state parameters at a time. Thus, individual measurements of the Measurement Equation (1) can usually be sorted into groups (blocks) of almost independent measurements. A Cluster Analysis software package was used. This is known as the method of *decoupling states*. However, the optimality of the high-pass filter would have been lost if any dependencies between the groups had been ignored. Fortunately, the error correlations between the measurement groups can be modelled as common regression parameters and/or estimated by Empirical Orthogonal Functions (EOF). These projection techniques include the multivariate statistical methods of Canonical Correlations and Discriminant Analysis, see Lange (1969a and 1969b), Rao (1975) and pages 11-13 of paper [4].

In paper [2], these "between groups" effects are taken into account as common calibration parameters $\mathbf{c}_t$ of a hybrid balloon tracking system. The matrix H_t of Augmented Model (9) is sparse and it can be partitioned accordingly. After discarding all non-informative equations, the large Canonical Block-Angular (CBA) equation system shrinks to the following sparse regression model:

$$\begin{bmatrix} \mathbf{y}_{t,1} \\ \mathbf{y}_{t,2} \\ \vdots \\ \mathbf{y}_{t,K} \\ \hat{\mathbf{c}}_{t-1} \end{bmatrix} = \begin{bmatrix} X_{t,1} & & & & G_{t,1} \\ & X_{t,2} & & & G_{t,2} \\ & & \ddots & & \vdots \\ & & & X_{t,K} & G_{t,K} \\ & & & & I \end{bmatrix} \begin{bmatrix} \mathbf{b}_{t,1} \\ \vdots \\ \mathbf{b}_{t,K} \\ \mathbf{c}_t \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{t,1} \\ \mathbf{e}_{t,2} \\ \vdots \\ \mathbf{e}_{t,K} \\ (\hat{\mathbf{c}}_{t-1} - \mathbf{c}_{t-1}) - \mathbf{a}_{t,c} \end{bmatrix} \qquad (11)$$

where vectors $\mathbf{b}_{t,1}$, $\mathbf{b}_{t,2}$,…, $\mathbf{b}_{t,K}$ represent information on the consecutive positions of a balloon that is being tracked during a sounding made at time t (t = 1, 2,…). The Jacobian matrices $X_{t,1}$, $X_{t,2}$,…, $X_{t,K}$ and $G_{t,1}$, $G_{t,2}$,…, $G_{t,K}$ represent the positional and the common calibration effects, respectively. The original system noise vector $\mathbf{a}_t = [\mathbf{a}_{t,1}', \mathbf{a}_{t,2}',\ldots, \mathbf{a}_{t,K}', \mathbf{a}_{t,c}']'$ contained the vector $\mathbf{a}_{t,c}$ of all calibration drifts.

Without loosing any generality, the measurement errors $\mathbf{e}_{t,1}$, $\mathbf{e}_{t,2}$,…, $\mathbf{e}_{t,K}$ in Equation (11) are now assumed to be orthonormalized separately within each group k (k = 1, 2,…, K). Thus, the optimal solution of a Kalman Filter (KF) for an overdetermined tracking system is written herein simply as follows:

$$\left.\begin{aligned} \hat{\mathbf{b}}_{t,k} &= (X_{t,k}'\, X_{t,k})^{-1} X_{t,k}' \left(\mathbf{y}_{t,k} - G_{t,k}\, \hat{\mathbf{c}}_t\right) \\ \hat{\mathbf{c}}_t &= \{ \sum_{k=0}^{K} G_{t,k}' \mathcal{R}_{t,k} G_{t,k} \}^{-1} \sum_{k=0}^{K} G_{t,k}' \mathcal{R}_{t,k} \mathbf{y}_{t,k} \end{aligned}\right\} \qquad (12)$$

where for t = 1, 2,… :

$\hat{\mathbf{c}}_t$ = estimated information on calibration for a sounding at time t

for k = 1, 2,…, K:

$\hat{\mathbf{b}}_{t,k}$ = estimated information on the k^{th} position of a balloon

$\mathcal{R}_{t,k} = I - X_{t,k} (X_{t,k}'\, X_{t,k})^{-1} X_{t,k}'$

and, for k = 0:

$G_{t,0} = I$

$\mathcal{R}_{t,0} = [\mathrm{Cov}(\hat{\mathbf{c}}_{t-1} - \mathbf{c}_{t-1}) + \mathrm{Cov}(\mathbf{a}_{t,c})]^{-1}$

$\mathbf{y}_{t,0} = \hat{\mathbf{c}}_{t-1}$

$\hat{\mathbf{c}}_0$ = estimated information on calibration at time 0

$\mathbf{c}_0$ = vector of initial calibration.

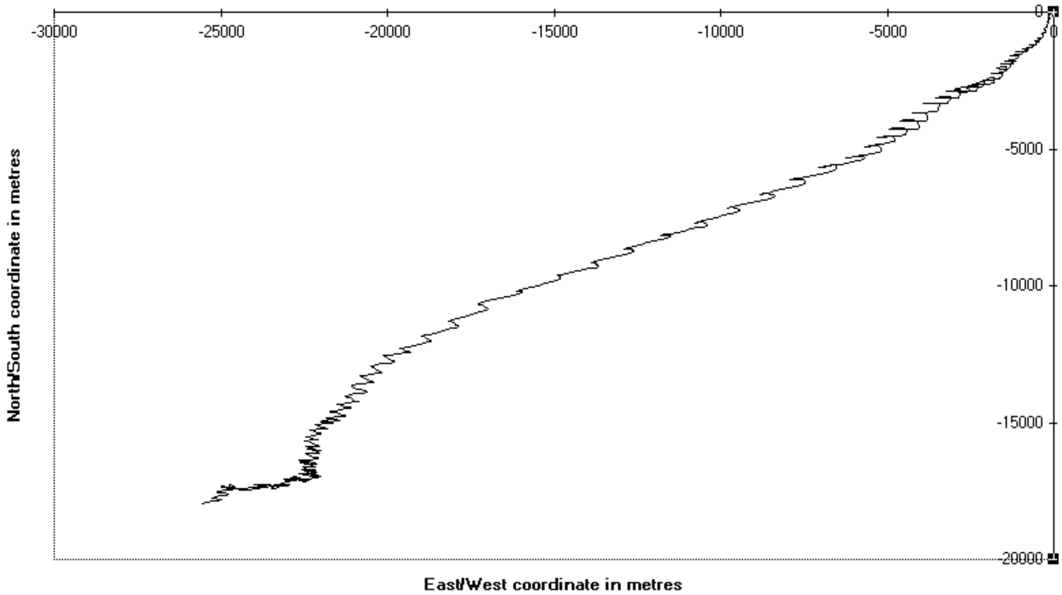
It is the index value k = 0 that makes Formula (12) the Fast Kalman Filtering (FKF) formula for the statistical calibration of sounding systems. Matrices $G_{t,0}$ and $\mathcal{R}_{t,0}$ represent the vehicle that conveys the following calibration information: $\mathbf{c}_t = \mathbf{c}_{t-1} + \mathbf{a}_{t,c}$; from a previous sounding t-1 to the Minimum Least Squares Estimation (MLSE) of the present sounding t. For t = 1, the estimate of $\mathbf{c}_0$ is given by $\hat{\mathbf{c}}_0$. This is a statistical *regularization* of the Measurement Equation system (1) that may otherwise be singular.

In paper [3], the FKF method has been generalized to the *Extended* Kalman Filtering (EKF). It is briefly described from the point of view of Numerical Weather Prediction (NWP) or, in fact, any large dynamical system. State vector $\mathbf{s}_t$ describes states of the atmosphere at time t (t = 1, 2,…). Grid values of various atmospheric variables e.g. the geopotential heights of different pressure levels are typically used. The dynamical behavior of the atmosphere is treated through linearizing the underlying partial differential equations of a NWP model and the local tangential approximation is used for a state transition matrix A_t. Thus, the System Equation (2) describes the time evolution of state parameters $\mathbf{s}_t$ (actually, their departure values from a "trajectory" in the *m*-dimensional space of state parameters) during an interval]t-1,t]. The state vectors $\mathbf{s}_t$ must contain data for a large amount of grid points for a realistic representation of the atmosphere and other state parameters. The latter parameters are related to the so-called physical parametrization schemes of small-scale atmospheric processes (i.e. identification of local linear models) and to systematic (calibration) errors of observing systems. The large inversion problem is basically overcome by decoupling states. Thus, the FKF solution is equivalent to the *Optimum Interpolation* (O/I) method of Gandin (1965) including an effective statistical calibration of observing systems and/or physical parametrization of the NWP model.

In paper $[4]$, the FKF method has been generalized to the *Adaptive* Kalman Filtering (AKF) where long *innovation sequences* (i.e. estimated measurement and system errors) are analysed in order to obtain improved estimates of different calibration or model parameters. Common factors F_t^y and F_t^s that may often be time-dependent are introduced here in order to make the error covariance matrices V_t (t = 1, 2,…) block- (or band-) diagonal for an extended form of the Augmented Model (9). Gross errors dA_t of the state transition matrices A_t (t = 1, 2,…) are first detected e.g. by spatial maximum correlation methods. These effects are then estimated here as "combined" regression parameters using a bilinear regression scheme based on the following identity: $d(A_t\mathbf{s}_t) \equiv dA_t\, \mathbf{s}_t + A_t\, d\mathbf{s}_t$. Early applications of these "combined" and "separate" regression parameters of Stratified Sampling were used in Lange (1965 and 1973). The FKF method makes it possible to process very large moving windows of data in real-time for improving the observability of different calibration and model parameters.

In paper $[5]$, a general model of a hybrid balloon tracking system is briefly described. Statistical calibration of different tracking sensors was made for a balloon tracking experiment in 1997. The iterated FKF computations were carried out by using a Hybrid Windfinding Algorithm (HWA). Figures 1 and 2 show the estimated maximum likelihood horizontal paths of the last balloon flight with and without the statistical calibration. The hybrid tracking computations were based on the two Loran-C signals of Slonim (8000-2) and Ejde (9007-M), whose bearing angles are almost perpendicular in Tikkakoski, Finland, as well as on the respective height, Optical (OT) and Radio Theodolite (RT) data.

Figure 1: The horizontal balloon path as computed from non-calibrated OT, RT and height data including the two Loran-C signals from Slonim and Ejde during the wind tracking of 20 July 1997 at 06 UTC



The emerged saw-tooth feature is not realistic in Figure 1. It results from systematic errors of angular measurements.

Figure 2: The horizontal balloon path as computed from FKF-calibrated OT, RT and height data including the two Loran-C signals from Slonim and Ejde during the wind tracking of 20 July 1997 at 06 UTC

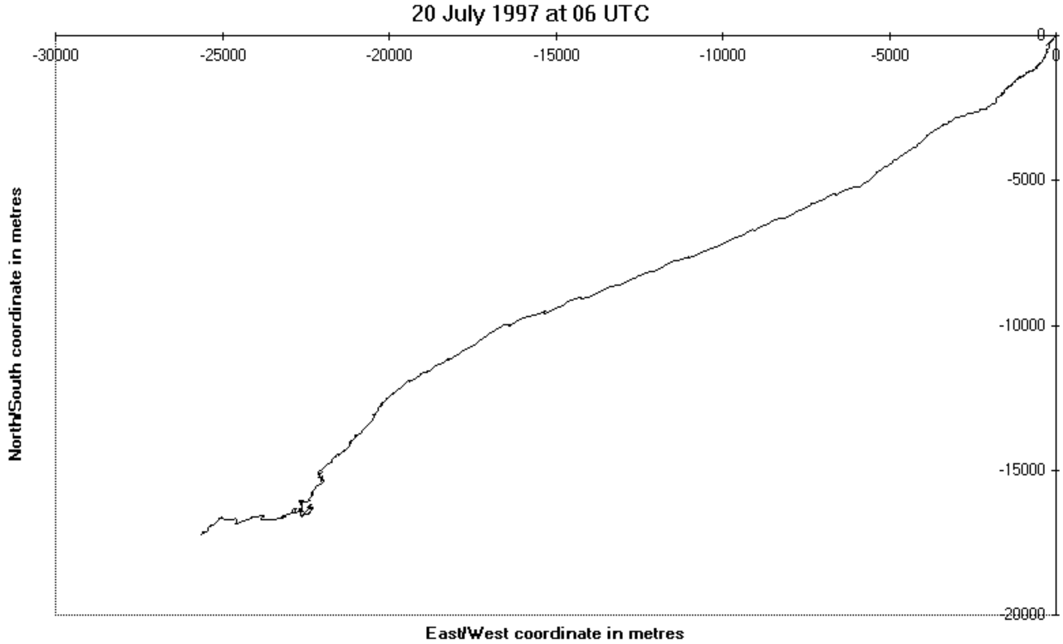


Figure 2 above exhibits a complete disappearance of the saw-tooth problem after the statistical calibration was made by the FKF method.

3. CONCLUDING REMARKS

The Fast Kalman Filter (FKF) algorithm specified in paper [2] can be used for estimating the systematic errors of a balloon tracking system by statistical means. There is no need for an expensive physical calibration if the system can be made overdetermined enough by using redundant signals or sensors. It is emphasized that a good optimality is necessary and that the stability of such FKF filtering processes should be monitored by computing the observation error covariances e.g. by MINQUE or AUE, see paper [5].

The FKF approach can obviously be generalized to the statistical calibration and accuracy estimation of other subsystems of the Global Observing System (GOS) of the World Weather Watch (WWW), see paper [1]. However, attempts to solve large Kalman Filters with the conventional Kalman (4) - (7) recursions are often doomed to fail due to inobservability problems, as the involved tedious numerical inversions prohibit the sizes of moving data windows from being large enough. By using the FKF method adaptively, as proposed in paper [4], these data windows may in some cases be made so large that neither a specific initialization nor a temporal training with long innovation sequences, as used often in real-time Kalman filtering applications, is necessary.

It is also emphasized that the entire GOS of the WWW is rather an underdetermined than an overdetermined observing system in many large areas of the Earth. Any erroneous calibration-related information that is repeatedly fed from operational numerical forecasts back to the observing systems, tends to become worse instead of disappearing. If a statistical calibration adjustment scheme is implemented then it must be understood that the whole process is a Kalman Filter. The FKF method helps to make it as close to the optimal as necessary. Thus, it should be used in combination with operational data-assimilation systems based on Global Circulation Models (GCM). Frequently made observing system intercomparisons are still needed for improving the observability of some calibration parameters that are weakly observable (Lange, 1988). All physically made calibration adjustments must also be reported for maintaining a good controllability.

A further application of optimal Kalman filtering using FKF could be data impact studies. Nowcasting and prediction accuracies of Numerical Weather Prediction (NWP) systems can be estimated, see paper [3]. Such impact studies are expected to demonstrate how the optimal density of an observing network varies depending on prevailing weather situations and social activities of communities. The theory of Kalman filtering, armed with the FKF and MINQUE/AUE computations, provides effective means for optimizing and controlling these observing networks in operational fashion.

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